Test 2A, Math 1410

Name: _____

PID Number: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will lose points, as will an answer with incorrect notation.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are allowed on this exam.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge and write your ID on both pages.
6. Number of questions = 10. Total Points = 100.

PID Number: _____

1. (8 points) Calculate y' if

$$y = \tan\left[\ln\left(\frac{1}{x^3}\right)\right]$$

2. (10 points) The law of Graham-Squire states that the following relationship holds between P (Pressure), V (Volume), and T (Temperature) of air inside a sphere. (Assume that P, V, and T are all positive numbers.)

$$\frac{PV^3}{T^2} = K$$

Where K is some constant.

(a) Assuming that the pressure is fixed (i.e. P is held constant), find the rate of change of the volume with respect to the temperature.

(b) Assuming that P is held constant, use your answer from (a) to explain why an increase in temperature will result in an increase in the volume. Hint: It may help to think about the volume function.

3. (14 points) For the equation $x^2 + xy + \sin^2 y = 1$, find $\dfrac{dy}{dx}$ and the equation for the tangent line at the point (1,0).

4. (12 points) Use calculus to find the x-coordinate(s) where $y = \dfrac{x^2 - 3}{x - 2}$ has a horizontal tangent line.

5. (8 points) Find $\frac{d}{dx}(\arccos(x)\sqrt{1-x^2})$. Simplify your answer.

6. (4 points) State if the following is true or false. Give a brief explanation to justify your answer.

   "The graph of $9x^2+4y^2=36$ is continuous and has no sharp corners or cusps, therefor the derivative $\dfrac{dy}{dx}$ exists for all points on the graph."

7. (10 points) Using only the derivatives for $\sin x$ and $\cos x$, as well as either the chain rule or the quotient rule, prove that

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

8. (10 points) Use differentials to approximate the change in the area of a circle as the radius changes from 5 cm to 5.3 cm.

9. (12 points) Let $A(x) = \dfrac{f(g(x))}{h(x)}$. Find $A'(0)$ if the following values hold for f, g, and h:

$$\begin{array}{ccc} f(0) = 4 & g(0) = 1 & h(0) = -2 \\ f(1) = -1 & g(1) = 6 & h(1) = 7 \\ f'(0) = -7 & g'(0) = 3 & h'(0) = 2 \\ f'(1) = -3 & g'(1) = -5 & h'(1) = 0 \end{array}$$

Simplify your answer. Note: You may not need all of the values above.

10. (12 points) Use logarithmic differentiation to calculate $f'(x)$ for

$$f(x) = (\cot x)^{1/x}$$

Simplify your answer.

Extra Credit (2 points): Is the following equation True or False? If true briefly explain why. If false, either explain why or give a counterexample.

$$\frac{d}{dx}[f(\sqrt{x})g(x)] = [f'(x) \cdot \frac{1}{2}x^{(-1/2)}]g(x) + g'(x)f(x)$$